M.Sc. / Sem I / Ist Paper

Abst. Algebra

Theorem Prove that if the quotient group G/H is simple, then the normal subgroup H of G is maximal.

Proof Given that G/H is a simple group.

~~Let~~ We have to prove that H is a maximal subgroup in G.

Let H is not maximal.

By definition of maximal subgroup, there exists a normal subgroup K of G such that $H \subset K \subset G$.

[Use of theorem]

- If G be a group, H be a normal subgroup of G and K i.e. a normal containing H, i.e. K then subgroup of G then K/H is a normal subgroup of G/H

$\Rightarrow$ K/H is a normal subgroup of G/H

Since $H \subset K \subset G$

i.e. H is a proper subset of K

and K is a proper subset of G,

$\Rightarrow$ K/H is a proper normal subgroup of G/H.

$\Rightarrow$ K/H is not equal to entire group G/H

and K/H is not equal to identity subgroup H/H.

$\Rightarrow$ G/H is not simple, because

a simple group has only two normal subgroups namely (i) itself and (ii) group having identity element only.

But, given that G/H is simple.

So, our supposition that H is not maximal, is wrong.

$\Rightarrow$ H is maximal.

(Recd)